Boundary-layer flow and heat transfer of non-Newtonian fluids in porous media

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Boundary-layer flow and heat transfer of non-Newtonian fluids in porous media are explored analytically. The local Nusselt number for forced and natural convection of non-Newtonian fluids in porous media on an isothermal semi-infinite plate is obtained as a function of the rheological parameters n and Ω. The results show that these parameters have a significant effect on the heat transfer rate and flow behavior.

Keywords: Porous media; non-Newtonian fluid; heat transfer; natural convection

Introduction

The theories used now to analyze the flow through a porous medium and to predict the heat transfer rates are based on the assumption that the fluid is Newtonian and hence Darcy's law holds. While for many fluids, such as water, air, and light oil, which are essential Newtonian fluids, this assumption is justified, it is not so for a large class of complex fluids, such as heavy crudes produced currently from certain oil fields, waxy crudes located in reservoirs of shallow depth, polymer solutions in chemical engineering, and blood. It is evident that the flow of a non-Newtonian fluid through a porous medium will have non-Newtonian characteristics.

Thus many works have been devoted to improving our understanding of isothermal flow of non-Newtonian fluids through a porous medium,[1–5] with diverse applications in petroleum reservoir engineering,[6,7] naval architecture,[8] polymer processing,[9] and lubrication in porous bearings.[10]

Identical to the isothermal flow of non-Newtonian fluids through porous media, nonisothermal flow has incurred in some important engineering applications; for example, enhanced recovery of heavy oil by thermal methods,[11] and polymer processing in packed beds.[12] Despite these applications and despite the strong interest expressed by the fluid mechanics and heat transfer communities in the same phenomenon without porous structure,[13] the nonisothermal flow and heat transfer of non-Newtonian fluids in porous media has escaped scrutiny.

For Newtonian fluids, viscosity is a parameter depending on temperature but not on conditions of measurement; non-Newtonian fluid viscosity, however, depends strongly on shear rate. In order to solve the problems associated with flow through porous media, an analytical expression of the curves of rheological behavior is required. Most of the frequently used empirical relations to express analytically the apparent viscosity in terms of shear rate are various forms of the power law. For example, the most widely used for a fluid with a structure at zero shear rate is the empirical equation

$$\tau = H\dot{\gamma}^n + \tau_0; \qquad \tau > \tau_0 \tag{1}$$

From this equation the apparent viscosity will be

$$\eta_a(\dot{\gamma}) = H\dot{\gamma}^{n-1} + \tau_0/\dot{\gamma} \tag{2}$$

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where n, H, and τ_0 are rheological parameters to be determined from rheological tests. These parameters depend strongly on temperature. Useful correlations for temperature dependence presented in the literature are[4]

$$H(T) = H(T_0)\exp[-a(T-T_0)/T_0] \tag{3}$$

$$\tau_0(T) = \tau_0(T_0) + b(T-T_0)/T_0 \tag{4}$$

and

$$n(T) = n(T_0) + c(T-T_0)/T_0 \tag{5}$$

in which T_0 is the reference temperature. The notations used are shown in the nomenclature.

From a rheological point of view the fluids for $n<1$ and $n>1$ are of pseudoplastic and dilatant type, respectively, with a reduction or increase in apparent viscosity as the shear rate increases. For $n=1$, relation 1 reduces to the well-known Bingham equation.

Obviously the rheological model described by Equation 1 reflects the two most important aspects of the behavior of non-Newtonian fluids observed from experimental data: the existence of a yield stress and non-Newtonian flow curve above the yield stress. At moderate and high shear rates the yield stress effect on apparent viscosity is insignificant. However, at low shear rate this effect becomes significant and cannot be neglected in Equation 2. For example, in the production of heavy crudes, a better rheological description should be the generalized Bingham model given by Equation 1, since the lower shear rates expected to be in the range 0.1–1.0 s^{-1} frequently occur in flow through porous media. However, another class of non-Newtonian fluids, such as polymer solutions and micro- and macroemulsions, which are often injected in oil reservoirs in order to improve the recovery efficiency, may be described by a power law equation without considering the yield stress.

From this discussion it is evident that an understanding of flow behavior of non-Newtonian fluids through porous media is essential for practical purposes. For nonisothermal flow one also must know the convective heat transfer processes from heated surfaces to the surrounding fluid-filled porous media with respect to evaluating the effect of buoyancy force due to temperature difference on flow, and for determining the temperature distribution corresponding to flow conditions so as to estimate the rheological parameters of the flow, since they depend strongly on temperature. The major objective of this investigation is to reveal the flow and heat transfer deviation from Newtonian behavior expressed in terms of rheological parameters, using very fundamental examples of boundary-layer flow, that is, forced and natural boundary-layer flows along an isothermal semi-infinite flat plate.

Basic equations

We develop the basic principles of flow mechanics and heat transfer through a non-Newtonian fluid-saturated porous medium. These principles mainly consist of mass conservation, a modified Darcy law, and energy conservation.

For a two-dimensional averaged flow of any fluid and relative to the coordinate system of Figure 1, the mass conservation statement reads

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

and the steady-state energy conservation has the form

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial x^2}\right) \tag{7}$$

where α is the thermal diffusivity.

To consider the rheological effect in the flow description through a porous medium, we need a modified Darcy law. For the rheological model given by Equation 1 the modified Darcy's law may be written as[4]

$$\mathbf{v}=\left[-\frac{K}{\mu_{\text{ef}}}(\text{grad}\, p-\boldsymbol{\alpha}_0)\right]^{1/n} \tag{8}$$

which is valid provided that

$$\begin{aligned}|\text{grad}\, p|&>\boldsymbol{\alpha}_0, \quad \mathbf{v}\neq 0\\ &\leqslant\boldsymbol{\alpha}_0, \quad \mathbf{v}=0\end{aligned} \tag{9}$$

Equation 8 was obtained by applying Kozeny's approach to a capillary model of a power law fluid in the presence of a yield stress. By this approach, the following expression for K/μ_{ef} was found:[14]

$$\frac{K}{\mu_{\text{ef}}}=\frac{1}{2H}\left(\frac{n\phi}{1+3n}\right)^n\left(\frac{8K}{\phi}\right)^{(1+n)/2} \tag{10}$$

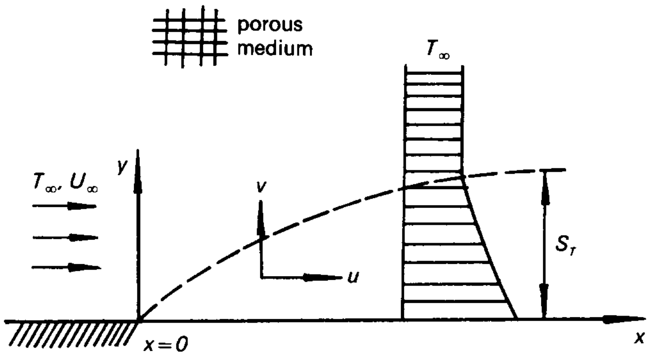

Figure 1 Forced convection boundary-layer flow through a porous medium near a heated horizontal wall

The threshold gradient in Equation 8 may be expressed in terms of yield stress:[3]

$$\alpha_0=\frac{\lambda\tau_0}{\sqrt{K}} \tag{11}$$

where ϕ is porosity and λ is a dimensionless constant to be determined experimentally. Most recently, the modified Darcy law (8) has been verified by experiment.[18]

It is obvious that when $n=1$ and $\tau_0=0$ for Newtonian fluids Equation 8 reduces to Darcy's law.

In the presence of a body force per unit volume ρg_x, Equation 8 becomes

$$\begin{aligned}&\mathbf{v}=\left(-\frac{K}{\mu_{\text{ef}}}(\text{grad}\, p+\rho g_x-\boldsymbol{\alpha}_0)\right)^{1/n} \quad \text{if } |\text{grad}\, p+\rho g_x|>\alpha_0\\ &\mathbf{v}=0 \quad \text{if } |\text{grad}\, p+\rho g_x|\leqslant\alpha_0\end{aligned} \tag{12}$$

acknowledging the fact that the flow through the porous medium stops when the externally controlled pressure gradient matches the hydrostatic gradient ρg_x.

Equations 6, 7, and 12 are composed of the governing equations for the flow and heat transfer of non-Newtonian fluids in porous media. To illustrate the rheological behavior effect on the flow and heat transfer in a porous medium, we consider two classical problems in convection processes through

Notation

a	Constant, Equation 3
b	Constant, Equation 4
c	Constant, Equation 5
f	Similarity stream function profile
g	Gravity
H	Consistency index
K	Permeability
L	Wall length
n	Power of non-Newtonian fluid
Nu	Nusselt number, Equation 40
p	Pressure
Ra_n	Modified Rayleigh number, Equation 35
T	Temperature
u, v	Velocity components, Figures 1 and 2
x, y	Cartesian coordinates, Figures 1 and 2

Greek symbols

α	Thermal diffusivity
α_0	Threshold gradient
β	Coefficient of thermal expansion
$\dot{\gamma}$	Shear rate
δ	Velocity boundary-layer thickness, Figure 2
δ_T	Temperature boundary-layer thickness, Figure 2
ΔT	Temperature difference, T_0-T_w
η	Similarity variable
η_a	Apparent viscosity
θ	Dimensionless temperature
λ	Constant
μ_{ef}	Effective viscosity
ρ	Density
τ	Shear stress
τ_0	Shear stress at zero shear rate
ϕ	Porosity
ψ	Stream function
Ω	Dimensionless rheological parameter, Equation 30

Subscripts

x	Local property
0	Wall property
∞	Porous reservoir property

porous media: forced and natural boundary-layer flows about an isothermal and semi-infinite plate.

Forced boundary layers

As a basic problem in heat transfer through porous media consisting of predicting the heat transfer rate between a differentially heated solid impermeable surface and a fluid-saturated porous medium, the forced boundary-layer parallel flow permeating through the porous material confined by the horizontal flat plate is considered, as seen in Figure 1.

Relative to the geometry and two-dimensional coordinate system of Figure 1, the steady-state governing equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{13}$$

$$u^n=\frac{K}{\mu_{ef}}\left[\left(-\frac{dp}{dx}\right)-\alpha_0\right] \quad \text{if} \left|-\frac{dp}{dx}\right|>\alpha_0$$
$$u=0 \quad \text{if} \left|-\frac{dp}{dx}\right|\leqslant\alpha_0 \tag{14}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{15}$$

where it has been assumed that ρ is constant, that the boundary layer is slender compared with the characteristic length L, and that the gravity effect is negligible.

Consider now the uniform parallel flow

$$u=U_\infty, \quad v=0, \quad p(x)=-\left(\frac{\mu_{ef}}{K}U_\infty^n+\alpha_0\right)x+\text{const.} \tag{16}$$

that satisfies the mass conservation (13) and momentum equations (14).

The heat transfer rates between the $x>0$ wall and the porous medium can be obtained by similarity solutions to the heat transfer problem described by Equations 13–15 and the boundary conditions of Figure 1. According to this similarity solution, the local Nusselt number is[16]

$$\text{Nu}_x=0.564\text{Pe}_x^{1/2} \tag{17}$$

if the local Peclet number is defined as

$$\text{Pe}_x=\frac{U_\infty x}{\alpha} \tag{18}$$

Averaging the heat transfer coefficient over the heated wall length L, we obtain

$$\text{Nu}_{0-L}=1.13\text{Pe}_L^{1/2} \tag{19}$$

Thus we conclude that the heat transfer correlation for forced boundary-layer flow of non-Newtonian fluids in porous media is the same as that for the flow of Newtonian fluids, if based on the velocity parameter U_∞. But they differ from each other when the dimensionless Peclet number is expressed on the basis of $-dp/dx$ as follows:

$$\text{Pe}_x=\left(-\frac{K}{\mu_{ef}}\frac{dp}{dx}\right)^{1/n}\frac{x}{\alpha}(1-\Omega)^{1/n} \tag{18$'$}$$

where

$$\Omega=\alpha_0/(-dp/dx) \tag{20}$$

Natural boundary layers

Consider the natural boundary-layer flow near a vertical impermeable surface embedded in a porous medium saturated

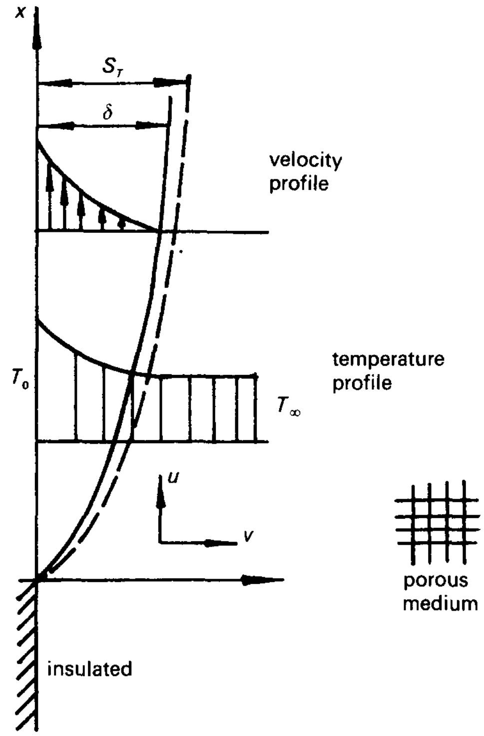

Figure 2 Natural convection boundary-layer flow through a porous medium near a vertical wall

with a non-Newtonian fluid (Figure 2). The surface is maintained at a constant temperature T_0 different from the porous medium temperature T_∞ sufficiently far from the wall. For this simplest boundary-layer problem for natural convection in porous media, the conservation equations for steady, constant-property free convective boundary-layer flow can be deduced from the basic equations presented above and written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{21}$$

$$u^n=\frac{K}{\mu_{ef}}\left(-\frac{dp}{dx}-\rho g_x-\alpha_0\right) \quad \text{if} \left|-\frac{\partial p}{\partial x}-\rho g_x\right|>\alpha_0$$
$$u=0 \quad \text{if} \left|-\frac{\partial p}{\partial x}-\rho g_x\right|\leqslant\alpha_0 \tag{22}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{23}$$

In Equations 21–23, ρ and μ_{ef} are the density and effective viscosity of the fluid; K is the intrinsic permeability of the porous medium.

The appropriate boundary conditions are

$$\begin{aligned} &v=0; \quad T=T_0 \quad &&\text{at } y=0,\ x>0 \\ &u\to 0; \quad T\to T_\infty \quad &&\text{as } y\to\infty \\ &u=0; \quad T=T_\infty \quad &&\text{at } x=0,\ y>0 \end{aligned} \tag{24}$$

For the free stream, Equation 22 gives

$$-\frac{\partial p}{\partial x}-\rho_\infty g=0 \tag{25}$$

Eliminating $\partial p/\partial x$ between Equations 22 and 25, we have

$$u^n = \frac{K}{\mu_{ef}}((\rho_\infty - \rho)g - \alpha_0) \quad \text{if } |(\rho_\infty - \rho)g| > \alpha_0$$
$$u = 0 \quad \text{if } |(\rho_\infty - \rho)g| \leqslant \alpha_0 \tag{26}$$

Taking into account the equation of state

$$\rho = \rho_\infty(1 - \beta(T - T_0)) \tag{27}$$

the Boussinesq-approximated momentum equation reduces to

$$u^n = \frac{\rho_\infty g \beta K}{\mu_{ef}}\left((T - T_\infty) - \frac{\alpha_0}{\rho_\infty g \beta}\right) \quad \text{if } \beta|T - T_\infty| > \frac{\alpha_0}{\rho_\infty g}$$
$$u = 0 \quad \text{if } \beta|T - T_\infty| \leqslant \frac{\alpha_0}{\rho_\infty g} \tag{28}$$

where β is the volumetric coefficient of thermal expansion.

Now we solve this boundary-layer heat transfer problem based on scale analysis, leaving the more accurate results of similarity analysis for presentation later.

The scales of the flow and temperature fields near the vertical surface of Figure 2 can be determined based on order-of-magnitude analysis. Consider for this purpose the thermal boundary layer of height L and thickness δ_T, and the velocity boundary layer of height L and thickness δ. For the flow of Newtonian fluids in porous media, $\delta = \delta_T$, which can be drawn from Darcy's law. However, for the flow of non-Newtonian fluids, the velocity profile will approach zero when T is equal to $T_\infty + \alpha_0/\rho_\infty g\beta$ at a specific point in the thermal boundary layer; thus $\delta \leqslant \delta_T$. This is illustrated mathematically in Equation 28 and schematically in Figure 2.

The velocity scale is, from Equation 28,

$$u \sim \frac{\rho_\infty g \beta K \,\Delta T}{\mu_{ef}}(1 - \Omega)^{1/n} \tag{29}$$

where

$$\Omega = \frac{\alpha_0}{g\rho_\infty \beta(T_0 - T_\infty)} \tag{30}$$

is a parameter related to the yield stress of non-Newtonian fluids.

In the velocity boundary layer (L, δ) the energy equation (Equation 10) indicates a balance between thermal diffusion from the side and vertical enthalpy flow:

$$u\frac{T(1-\Omega)}{L} \quad \text{or} \quad v\frac{T(1-\Omega)}{\delta} \sim \alpha\frac{T(1-\Omega)}{\delta^2} \tag{31}$$

Since the principle of mass conservation (21) in the (L, δ) layer requires

$$u/L \sim v/\delta$$

we realize that the two convection scales in Equation 31 are of the same order of magnitude; hence, energy conservation in the velocity boundary layer requires

$$u\frac{T(1-\Omega)}{L} \sim \alpha\frac{T(1-\Omega)}{\delta^2} \tag{32}$$

or

$$\delta \sim L\,\mathrm{Ra}_n^{-1/2}(1-\Omega)^{-1/2n} \tag{33}$$

$$u \sim \alpha/L\,\mathrm{Ra}_n \tag{34}$$

where Ra_n is defined similar to the Darcy-modified Rayleigh number used routinely in natural convection heat transfer through porous media, and it includes the rheological behavior of non-Newtonian flow:

$$\mathrm{Ra}_n = \left(\frac{\rho_\infty g \beta K(T_0 - T_\infty)}{\mu_{ef}}\right)^{1/n}\frac{L}{\alpha} \tag{35}$$

The scale of thermal boundary-layer thickness δ_T can be obtained by integrating the energy conservation equation, Equation 23, in the thermal boundary (L, δ_T); that is,

$$\frac{d}{dx}\int_0^{\delta_T} u(T - T_\infty)\,dy = -\alpha\left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{36}$$

Since $u = 0$ in the region $\delta \leqslant y \leqslant \delta_T$, the above integral can be simplified to

$$\frac{d}{dx}\int_0^{\delta} u(T - T_\infty)\,dy = -\alpha\left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{37}$$

The scaling equivalence recommended by Equation 37 is

$$\frac{u\delta}{L}\Delta T(1-\Omega) \sim \alpha\frac{\Delta T}{\delta_T} \tag{38}$$

where the u scale is dictated by the δ-layer scale (Equation 34). Combining, we find

$$\delta_T \sim \frac{1}{1-\Omega}\delta \sim L\,\mathrm{Ra}_n^{-1/2}(1-\Omega)^{-(2n+1)/2n} \tag{39}$$

The overall Nusselt number

$$\mathrm{Nu} = q/k\,\Delta T \tag{40}$$

scales as

$$\mathrm{Nu} \sim \frac{(kL\,\Delta T/\delta_T)}{k\,\Delta T} \sim \mathrm{Ra}_n^{1/2}(1-\Omega)^{(2n+1)/2n} \tag{41}$$

The flow field and heat transfer scales in Equations 33, 34, 39, and 41 agree within a factor of order one with the similarity solution, as we shall see.

Natural boundary layers of non-Newtonian fluids in porous media have two length scales, δ and δ_T. This feature distinguishes them from their counterparts in Newtonian fluids, which are characterized by a single length scale δ_T.

The similarity formulation of the isothermal wall problem starts with deducing the similarity variable from Equations 33 and 39:

$$\eta = \frac{y}{x}\mathrm{Ra}_{n,x}^{1/2} \tag{42}$$

Introducing the similarity profiles as for Newtonian fluids gives

$$\frac{\psi}{\alpha\,\mathrm{Ra}_{n,x}^{1/2}} = f(\eta), \qquad \frac{T - T_\infty}{T_0 - T_\infty} = \theta(\eta) \tag{43}$$

The problem statement (20–23) becomes

$$f' = (\theta - \Omega)^{1/n} \quad \text{if } \theta > \Omega$$
$$f' = 0 \quad \text{if } \theta \leqslant \Omega \tag{44}$$

$$\theta'' + \tfrac{1}{2}f\theta' = 0 \tag{45}$$

$$f(0) = 0, \qquad \theta(0) = 1$$
$$f'(\infty) = 0, \qquad \theta(\infty) = 0 \tag{46}$$

The problem stated as Equations 44–46 was solved numerically for a wide range of the rheological parameters n and Ω. The numerical procedure was the standard "shooting" method, by which Equations 44–46 were integrated from $\eta = 0$ onward, using the fourth-order Runge-Kutta method. To initiate the integration, we had to guess the value of $\theta'(0)$ and to adjust this guess until the outer boundary condition (46) was satisfied. The numerical results presented in this paper were all obtained based on the shooting success criterion

$$|\theta(\infty)| \leqslant 5 \times 10^{-4} \tag{47}$$

A shooting distance $\eta \geqslant 15$ was found to be adequate in simulating the $\eta \to \infty$ limit of Equation 46.

The local Nusselt number can be expressed by

$$\mathrm{Nu}_x = -\theta'(0)\,\mathrm{Ra}_{n,x}^{1/2} \tag{48}$$

where

$$\mathrm{Ra}_{n,x} = \left(\frac{\rho_\infty g \beta K (T_0 - T_\infty)}{\mu_{\mathrm{ef}}}\right)^{1/n} \frac{x}{\alpha} \tag{49}$$

Obviously, the value $-\theta'(0)$ depends on the rheological parameters of non-Newtonian fluids n and Ω, and is expressed as

$$-\theta'(0) = F(n, \Omega) \tag{50}$$

The value of $-\theta'(0)$ approaches a constant when $n=1$ and $\Omega=0$. In fact, Table 1 shows that this limiting constant is

$$F(1, 0) = 0.444 \tag{51}$$

which agrees with the earlier calculation of Cheng and Minkowycz[17] for the local Nusselt number for a Newtonian flow through porous media.

The local Nusselt number is listed in Table 1 for a wide range of parameters n and Ω. These values agree to within a numerical factor of order 1 with the overall scales discussed above. Below we discuss the similarity results by focusing individually on the two rheological effects that deviate the flow and heat transfer phenomena from the Newtonian flow and heat transfer through porous media, namely, the effect of power n and the parameter Ω which result from the yield stress.

The effect of rheological parameters n and Ω

Figures 3–5 show the effects of the rheological parameters n and Ω on the dimensionless temperature and velocity profiles. These plots indicate a significant deviation existing between Newtonian flows and non-Newtonian flows. For example, for a given value of the dimensionless group $\Omega = \alpha_0/\rho_\infty g \beta\, \Delta T$ (i.e., the flow in the presence of a threshold gradient), the temperature for a pseudoplastic fluid is much larger than that of Newtonian fluids, while the velocity is smaller. The trend for a dilatant fluid is just negative. On the other hand, for a given value of rheological parameter n, Figures 3–5 indicate that the threshold gradient has a significant effect on the temperature distribution as well as on flow behavior for a power law fluid. Temperature profiles become flatter and flatter, and the maximum velocities in the boundary layer decrease as the parameter Ω increases.

The thermal boundary layer thickness δ_T also depends strongly on the rheological parameters n and Ω. It has been seen from Figures 3–5 that δ_T increases as n decreases and/or Ω increases. Thus, the thermal boundary layer is much thicker for a pseudoplastic fluid with a threshold gradient than one in

Table 1 Summary of similarity solutions for local Nusselt number $-\theta'(0)$

n \ Ω	0.0	0.2	0.4	0.6	0.8	1.0
0.4	0.353	0.244	0.156	0.0965	0.0699	0.0
0.8	0.424	0.338	0.255	0.168	0.0981	0.0
1.0	0.444	0.365	0.282	0.196	0.115	0.0
1.2	0.459	0.386	0.305	0.218	0.130	0.0
1.5	0.475	0.409	0.332	0.245	0.150	0.0

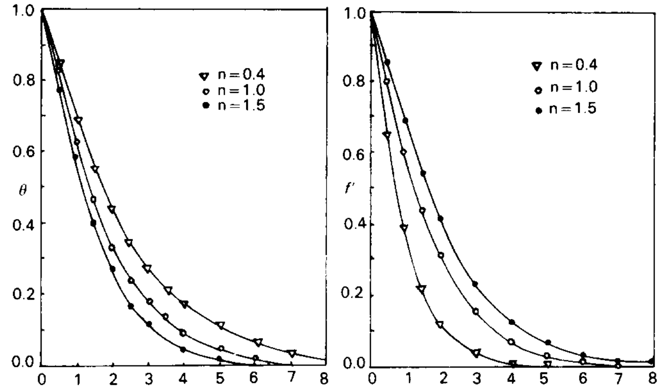

Figure 3 The effect of power n when $\Omega=0.0$: (a) temperature profiles; (b) velocity profiles

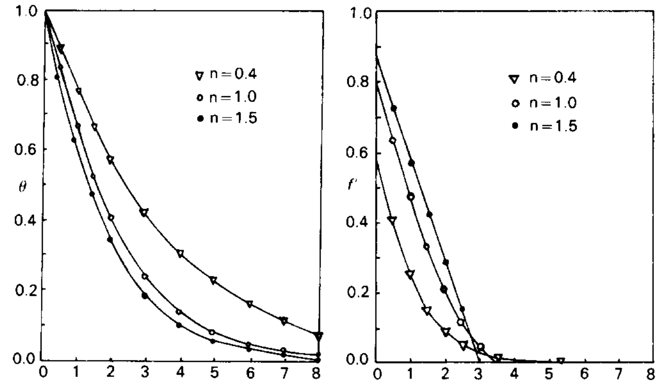

Figure 4 The effect of power n when $\Omega=0.2$: (a) temperature profiles; (b) velocity profiles

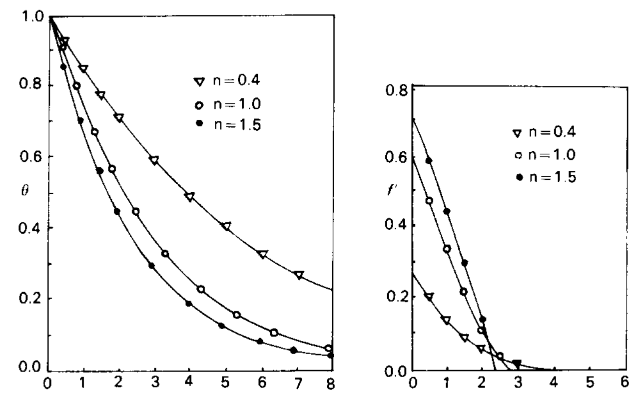

Figure 5 The effect of power n when $\Omega=0.4$: (a) temperature profiles; (b) velocity profiles

Newtonian flow. Due to the direct relation between local Nusselt number and thermal boundary-layer thickness, Nu is surely affected by n and Ω. This has been illustrated by the scale results and similarity results in Table 1. Nu_x can be approximately correlated as a function of the rheological parameters n and Ω in the form

$$\frac{\mathrm{Nu}_x}{\mathrm{Ra}_{n,x}^{1/2}} = \sqrt{\frac{n}{3n+2}}\,(1-\Omega)^{1/2n}\left(1-\frac{\Omega}{2}\right) \tag{52}$$

This correlation predicts Nu within 10% accuracy for the parameter range of $0.5 \leqslant n \leqslant 1.5$ and $0 \leqslant \Omega \leqslant 1$. From this equation or Table 1, we see that the threshold gradient manifested in the dimensionless group Ω has more significant effects on Nu for a pseudoplastic fluid as compared with a dilatant fluid.

For power law fluids with no yeld stress (that is, $\Omega=0$), the correlation 52 is simplified to

$$\frac{\mathrm{Nu}_x}{\mathrm{Ra}_{n,x}^{1/2}}=\sqrt{\frac{n}{3n+2}} \tag{53}$$

This equation has been derived by the integral method for power law fluids[19] and predicts Nu within 1% accuracy for $0.4 \leqslant n \leqslant 1.5$.

Summary

Boundary-layer flow and heat transfer of non-Newtonian fluids in porous media has been explored analytically. For a non-isothermal flow, heat transfer knowledge is required to evaluate the effect of the buoyancy force due to temperature differences on fluid flow and to specify the temperature distribution for the calculation of the rheological parameters n, τ_0, and H.

The results obtained by the investigation on flow and heat transfer in forced and natural boundary layers on an isothermal semi-infinite plate revealed that n, τ_0, and H have a significant effect on the heat transfer rate and flow behavior through a porous medium. These rheological parameters are associated with physical properties of the fluid and the porous medium, and boundary conditions of the problems in several dimensionless groups. For example, for natural convection of non-Newtonian fluids on a vertical isothermal plate the dimensionless parameter

$$\mathrm{Ra}_n=\left(\frac{\rho_\infty \beta g K\,\Delta T}{\mu_{\mathrm{ef}}}\right)^{1/n}\frac{L}{\alpha}$$

extends the application of the Darcy-modified Rayleigh number in Newtonian flow through a porous medium to the non-Newtonian case, and the dimensionless group

$$\Omega=\alpha_0/\rho_\infty g\beta\,\Delta T$$

determines the influence of threshold gradient on heat transfer and flow behavior. If the flow is produced by an externally forced pressure gradient, this group becomes

$$\Omega=\alpha_0/(-\mathrm{d}p/\mathrm{d}x)$$

The heat transfer correlation for forced convection of non-Newtonian fluids in porous media was found to be the same as that for Newtonian fluids,

$$\mathrm{Nu}_x=0.564\mathrm{Pe}_x^{1/2}$$

if the Peclet number is defined based on the free stream velocity U_∞, but not on pressure gradient. The heat transfer correlation for natural convection is approximately expressed by

$$\frac{\mathrm{Nu}_x}{\mathrm{Ra}_{n,x}^{1/2}}=\sqrt{\frac{n}{3n+2}}(1-\Omega)^{1/2n}\left(1-\frac{\Omega}{2}\right)$$

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